Chapter 1

Algebraic Expressions

Exercise 1A SKILLS INTERPRETATION

1 Simplify these expressions:

a $x^3 \times x^4$ **b** $2x^3 \times 3x^2$ **c** $\frac{k^3}{k^2}$

d $\frac{4p^3}{2p}$ **e** $\frac{3x^3}{3x^2}$ **f** $(y^2)^5$

g $10x^5 \div 2x^3$ **h** $(p^3)^2 \div p^4$ **i** $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$ **k** $2a^4 \times 3a^5$ **l** $\frac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$ **n** $3x^3 \times 2x^2 \times 4x^6$ **o** $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$ **q** $2a^3 \div 3a^2 \times 6a^5$ **r** $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x - 2)$ **b** $x(x + 9)$ **c** $-3y(4 - 3y)$

d $x(y + 5)$ **e** $-x(3x + 5)$ **f** $-5x(4x + 1)$

g $(4x + 5)x$ **h** $-3y(5 - 2y^2)$ **i** $-2x(5x - 4)$

j $(3x - 5)x^2$ **k** $3(x + 2) + (x - 7)$ **l** $5x - 6 - (3x - 2)$

m $4(c + 3d^2) - 3(2c + d^2)$ **n** $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

o $x(3x^2 - 2x + 5)$ **p** $7y^2(2 - 5y + 3y^2)$ **q** $-2y^2(5 - 7y + 3y^2)$

r $7(x - 2) + 3(x + 4) - 6(x - 2)$ **s** $5x - 3(4 - 2x) + 6$

t $3x^2 - x(3 - 4x) + 7$ **u** $4x(x + 3) - 2x(3x - 7)$ **v** $3x^2(2x + 1) - 5x^2(3x - 4)$

3 Simplify these fractions:

a $\frac{6x^4 + 10x^6}{2x}$ **b** $\frac{3x^5 - x^7}{x}$ **c** $\frac{2x^4 - 4x^2}{4x}$

d $\frac{8x^3 + 5x}{2x}$ **e** $\frac{7x^7 + 5x^2}{5x}$ **f** $\frac{9x^5 - 5x^3}{3x}$

Exercise 1B SKILLS INTERPRETATION

1 Expand and simplify if possible:

a $(x + 4)(x + 7)$ **b** $(x - 3)(x + 2)$ **c** $(x - 2)^2$

d $(x - y)(2x + 3)$ **e** $(x + 3y)(4x - y)$ **f** $(2x - 4y)(3x + y)$

g $(2x - 3)(x - 4)$ **h** $(3x + 2y)^2$ **i** $(2x + 8y)(2x + 3)$

j $(x + 5)(2x + 3y - 5)$ **k** $(x - 1)(3x - 4y - 5)$ **l** $(x - 4y)(2x + y + 5)$

m $(x + 2y - 1)(x + 3)$ **n** $(2x + 2y + 3)(x + 6)$ **o** $(4 - y)(4y - x + 3)$

p $(4y + 5)(3x - y + 2)$ **q** $(5y - 2x + 3)(x - 4)$ **r** $(4y - x - 2)(5 - y)$

2 Expand and simplify if possible:

a $5(x + 1)(x - 4)$ **b** $7(x - 2)(2x + 5)$ **c** $3(x - 3)(x - 3)$

d $x(x - y)(x + y)$ **e** $x(2x + y)(3x + 4)$ **f** $y(x - 5)(x + 1)$

g $y(3x - 2y)(4x + 2)$ **h** $y(7 - x)(2x - 5)$ **i** $x(2x + y)(5x - 2)$

j $x(x + 2)(x + 3y - 4)$ **k** $y(2x + y - 1)(x + 5)$ **l** $y(3x + 2y - 3)(2x + 1)$

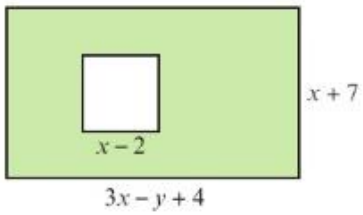
m $x(2x + 3)(x + y - 5)$ **n** $2x(3x - 1)(4x - y - 3)$ **o** $3x(x - 2y)(2x + 3y + 5)$

p $(x + 3)(x + 2)(x + 1)$ **q** $(x + 2)(x - 4)(x + 3)$ **r** $(x + 3)(x - 1)(x - 5)$

s $(x - 5)(x - 4)(x - 3)$ **t** $(2x + 1)(x - 2)(x + 1)$ **u** $(2x + 3)(3x - 1)(x + 2)$

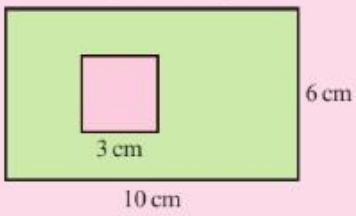
v $(3x - 2)(2x + 1)(3x - 2)$ **w** $(x + y)(x - y)(x - 1)$ **x** $(2x - 3y)^3$

(P) **3** The diagram shows a rectangle with a square cut out.
The rectangle has length $3x - y + 4$ and width $x + 7$.
The square has side length $x - 2$.
Find an expanded and simplified expression for the area **shaded** green.



Problem-solving

Use the same strategy as you would use if the lengths were given as numbers:



(P) **4** A **cuboid** has **dimensions** $(x + 2)$ cm, $(2x - 1)$ cm and $(2x + 3)$ cm.
Show that the volume of the cuboid is $(4x^3 + 12x^2 + 5x - 6)$ cm^3.

(E/P) **5** **Given that** $(2x + 5y)(3x - y)(2x + y) = ax^3 + bx^2y + cxy^2 + dy^3$, where a, b, c and d are **constants**, find the values of a, b, c and d. **(2 marks)**

Challenge

Expand and simplify $(x + y)^4$.

Exercise 1C SKILLS PROBLEM-SOLVING

1 Factorise these expressions completely:

a $4x + 8$ **b** $6x - 24$ **c** $20x + 15$

d $2x^2 + 4$ **e** $4x^2 + 20$ **f** $6x^2 - 18x$

g $x^2 - 7x$ **h** $2x^2 + 4x$ **i** $3x^2 - x$

j $6x^2 - 2x$ **k** $10y^2 - 5y$ **l** $35x^2 - 28x$

m $x^2 + 2x$ **n** $3y^2 + 2y$ **o** $4x^2 + 12x$

p $5y^2 - 20y$ **q** $9xy^2 + 12x^2y$ **r** $6ab - 2ab^2$

s $5x^2 - 25xy$ **t** $12x^2y + 8xy^2$ **u** $15y - 20yz^2$

v $12x^2 - 30$ **w** $xy^2 - x^2y$ **x** $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$ **b** $2x^2 + 6x$ **c** $x^2 + 11x + 24$

d $x^2 + 8x + 12$ **e** $x^2 + 3x - 40$ **f** $x^2 - 8x + 12$

g $x^2 + 5x + 6$ **h** $x^2 - 2x - 24$ **i** $x^2 - 3x - 10$

j $x^2 + x - 20$ **k** $2x^2 + 5x + 2$ **l** $3x^2 + 10x - 8$

m $5x^2 - 16x + 3$ **n** $6x^2 - 8x - 8$

o $2x^2 + 7x - 15$ **p** $2x^4 + 14x^2 + 24$

q $x^2 - 4$ **r** $x^2 - 49$

s $4x^2 - 25$ **t** $9x^2 - 25y^2$ **u** $36x^2 - 4$

v $2x^2 - 50$ **w** $6x^2 - 10x + 4$ **x** $15x^2 + 42x - 9$

Hint For part **n**, take 2 out as a common factor first. For part **p**, let $y = x^2$.

3 Factorise completely:

a $x^3 + 2x$ **b** $x^3 - x^2 + x$ **c** $x^3 - 5x$

d $x^3 - 9x$ **e** $x^3 - x^2 - 12x$ **f** $x^3 + 11x^2 + 30x$

g $x^3 - 7x^2 + 6x$ **h** $x^3 - 64x$ **i** $2x^3 - 5x^2 - 3x$

j $2x^3 + 13x^2 + 15x$ **k** $x^3 - 4x$ **l** $3x^3 + 27x^2 + 60x$

E/P **4** Factorise completely $x^4 - y^4$. **(2 marks)**

Problem-solving

Watch out for terms that can be written as a function of a function, for example: $x^4 = (x^2)^2$.

E **5** Factorise completely $6x^3 + 7x^2 - 5x$. **(2 marks)**

Challenge

Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

Exercise 1D SKILLS PROBLEM-SOLVING

1 Simplify:

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{5}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{\frac{7}{3}} \div x^{\frac{2}{3}}$ **i** $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$ **k** $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$ **l** $\frac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate, without using your calculator:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{3}{2}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$ **h** $1296^{\frac{3}{4}}$ **i** $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ **k** $\left(\frac{6}{5}\right)^{-1}$ **l** $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\frac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\frac{x + 4x^3}{x^3}$

e $\frac{2x + x^2}{x^4}$ **f** $\left(\frac{4}{9}x^4\right)^{\frac{3}{2}}$ **g** $\frac{9x^2 - 15x^5}{3x^3}$ **h** $\frac{5x + 3x^2}{15x^3}$

(E) **4 a** Find the value of $81^{\frac{1}{4}}$. **(1 mark)**

b Simplify $x(2x^{-\frac{1}{3}})^4$. **(2 marks)**

(E) **5** Given that $y = \frac{1}{8}x^3$, express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(2 marks)**

b $\frac{1}{2}y^{-2}$ **(2 marks)**

Exercise 1E SKILLS PROBLEM-SOLVING

Do not use your calculator for this exercise.

1 Simplify:

a $\sqrt{28}$ **b** $\sqrt{72}$ **c** $\sqrt{50}$

d $\sqrt{32}$ **e** $\sqrt{90}$ **f** $\frac{\sqrt{12}}{2}$

g $\frac{\sqrt{27}}{3}$ **h** $\sqrt{20} + \sqrt{80}$ **i** $\sqrt{200} + \sqrt{18} - \sqrt{72}$

j $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$ **k** $\sqrt{28} - 2\sqrt{63} + \sqrt{7}$ **l** $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

m $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$ **n** $\frac{\sqrt{44}}{\sqrt{11}}$ **o** $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2 + \sqrt{3})$ **b** $\sqrt{5}(3 - \sqrt{3})$ **c** $\sqrt{2}(4 - \sqrt{5})$

d $(2 - \sqrt{2})(3 + \sqrt{5})$ **e** $(2 - \sqrt{3})(3 - \sqrt{7})$ **f** $(4 + \sqrt{5})(2 + \sqrt{5})$

g $(5 - \sqrt{3})(1 - \sqrt{3})$ **h** $(4 + \sqrt{3})(2 - \sqrt{3})$ **i** $(7 - \sqrt{11})(2 + \sqrt{11})$

(E) **3** Simplify $\sqrt{75} - \sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer. **(2 marks)**

Exercise 1F SKILLS ANALYSIS

Do not use your calculator for this exercise.

1 Simplify:

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$ **c** $\frac{1}{\sqrt{2}}$ **d** $\frac{\sqrt{3}}{\sqrt{15}}$

e $\frac{\sqrt{12}}{\sqrt{48}}$ **f** $\frac{\sqrt{5}}{\sqrt{80}}$ **g** $\frac{\sqrt{12}}{\sqrt{156}}$ **h** $\frac{\sqrt{7}}{\sqrt{63}}$

2 Rationalise the denominators and simplify:

a $\frac{1}{1+\sqrt{3}}$ **b** $\frac{1}{2+\sqrt{5}}$ **c** $\frac{1}{3-\sqrt{7}}$ **d** $\frac{4}{3-\sqrt{5}}$ **e** $\frac{1}{\sqrt{5}-\sqrt{3}}$

f $\frac{3-\sqrt{2}}{4-\sqrt{5}}$ **g** $\frac{5}{2+\sqrt{5}}$ **h** $\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ **i** $\frac{11}{3+\sqrt{11}}$ **j** $\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

k $\frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$ **l** $\frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$ **m** $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

3 Rationalise the denominators and simplify:

a $\frac{1}{(3-\sqrt{2})^2}$ **b** $\frac{1}{(2+\sqrt{5})^2}$ **c** $\frac{4}{(3-\sqrt{2})^2}$

d $\frac{3}{(5+\sqrt{2})^2}$ **e** $\frac{1}{(5+\sqrt{2})(3-\sqrt{2})}$ **f** $\frac{2}{(5-\sqrt{3})(2+\sqrt{3})}$

E/P **4** Simplify $\frac{3-2\sqrt{5}}{\sqrt{5}-1}$ giving your answer in the form $p+q\sqrt{5}$, where p and q are rational numbers. **(4 marks)**

Problem-solving

You can check that your answer is in the correct form by writing down the values of p and q and checking that they are rational numbers.

Chapter review 1 SKILLS EXECUTIVE FUNCTION

1 Simplify:

a $y^3 \times y^5$ **b** $3x^2 \times 2x^5$ **c** $(4x^2)^3 \div 2x^5$ **d** $4b^2 \times 3b^3 \times b^4$

2 Expand and simplify if possible:

a $(x+3)(x-5)$ **b** $(2x-7)(3x+1)$ **c** $(2x+5)(3x-y+2)$

3 Expand and simplify if possible:

a $x(x+4)(x-1)$ **b** $(x+2)(x-3)(x+7)$ **c** $(2x+3)(x-2)(3x-1)$

4 Expand the brackets:

a $3(5y+4)$ **b** $5x^2(3-5x+2x^2)$ **c** $5x(2x+3)-2x(1-3x)$ **d** $3x^2(1+3x)-2x(3x-2)$

5 Factorise these expressions completely:

a $3x^2 + 4x$ **b** $4y^2 + 10y$ **c** $x^2 + xy + xy^2$ **d** $8xy^2 + 10x^2y$

6 Factorise:

a $x^2 + 3x + 2$ **b** $3x^2 + 6x$ **c** $x^2 - 2x - 35$ **d** $2x^2 - x - 3$

e $5x^2 - 13x - 6$ **f** $6 - 5x - x^2$

7 Factorise:

a $2x^3 + 6x$ **b** $x^3 - 36x$ **c** $2x^3 + 7x^2 - 15x$

8 Simplify:

a $9x^3 \div 3x^{-3}$ **b** $(4^{\frac{3}{2}})^{\frac{1}{3}}$ **c** $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

9 Evaluate, without using your calculator:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **b** $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify, without using your calculator:

a $\frac{3}{\sqrt{63}}$ **b** $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

11 **a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.

b By factorising the expression, show that your answer to part **a** can be written as the product of two **prime** factors.

12 Expand and simplify if possible, without using your calculator:

a $\sqrt{2}(3 + \sqrt{5})$ **b** $(2 - \sqrt{5})(5 + \sqrt{3})$ **c** $(6 - \sqrt{2})(4 - \sqrt{7})$

13 Rationalise the denominator and simplify:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{\sqrt{2} - 1}$ **c** $\frac{3}{\sqrt{3} - 2}$ **d** $\frac{\sqrt{23} - \sqrt{37}}{\sqrt{23} + \sqrt{37}}$ **e** $\frac{1}{(2 + \sqrt{3})^2}$ **f** $\frac{1}{(4 - \sqrt{7})^2}$

14 **Do not use your calculator for this question.**

a Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.

b **Hence**, fully factorise $x^3 - x^2 - 17x - 15$.

(E) **15** Given that $y = \frac{1}{64}x^3$, express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(1 mark)**

b $4y^{-1}$ **(1 mark)**

(E/P) **16** Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(5 marks)**

(E) **17** Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$, without using your calculator. **(2 marks)**

(E) **18** Factorise completely $x - 64x^3$. **(3 marks)**

(E/P) **19** Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

E/P **20** Solve the equation $8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$.

Give your answer in the form $a\sqrt{b}$, where a and b are integers. **(4 marks)**

P **21** **Do not use your calculator for this question.**
A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm^2.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

E **22** Show that $\frac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

E/P **23** Given that $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

E/P **24** Given that $\frac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$,
write down the value of a and the value of b. **(2 marks)**

Challenge

a Simplify $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$.

b Hence show that $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \ldots + \frac{1}{\sqrt{24} + \sqrt{25}} = 4$

Summary of key points

1 You can use the laws of indices to simplify powers of the same base.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$

2 Factorising is the opposite of expanding brackets.

3 A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.

4 $x^2 - y^2 = (x + y)(x - y)$

5 You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$

6 You can manipulate surds using these rules:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

7 The rules to rationalise denominators are:

- For fractions in the form $\frac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
- For fractions in the form $\frac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $(a - \sqrt{b})$.
- For fractions in the form $\frac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $(a + \sqrt{b})$.